Hall Ticket Number:

Code No. : 1116 I

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. I Year I-Semester Examinations*, July/August-2016

Mathematics-I

Time: 3 hours | Max. Marks: 70

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

1. Define Rank of a matrix.
2. Find the area of quadrant of the circle $x^2 + y^2 = 1$ in the first quadrant using double integration.
3. Give an example of a series which is conditionally convergent.
4. Write the necessary and sufficient conditions for a function of two variables f(x,y) to have maximum or minimum.
5. Define the Envelope of a curve.
6. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2+3}{n^2+5}$
7. Define the Jacobins in Cartesian coordinates in two dimensions.
8. Find the Eigen values of matrix $A = \begin{bmatrix} 4 & 3 \\ 2 & 9 \end{bmatrix}$
9. Evaluate $\int_0^1 \int_0^x e^{y/x}\, dy\, dx$
10. Expand the function $sin^{-1}x$ in powers of x upto three terms.

Part-B (5 × 10 = 50 Marks)
(All bits carry equal marks)

11. a) Find the Eigen values and Eigen Vectors of $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

    b) Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to canonical form by Orthogonal transformation.

12. a) Test the convergence $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots\ldots\ldots + \frac{x^n}{n^2+1} \ldots\ldots$

    b) Prove that the series $\frac{\sin x}{1^3} - \frac{\sin 2x}{2^3} + \frac{\sin 3x}{3^3} - \cdots$ converges absolutely.

13. a) Find the radius of curvature at the point (a,0) of the curve $y^3x = a^3 - x^3$

    b) Find the evolute of the parabola $y^2 = 4ax$.

14. a) If $u(x,y) = x^2 Tan^{-1}\left(\frac{y}{x}\right) - y^2 Tan^{-1}\left(\frac{x}{y}\right), x > 0, y > 0$, then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$

    b) Find the maximum and minimum values of the function $f(x,y) = x^3 + y^3 - 3axy$

Contd…2

15. a) Change the order of integration $\int_0^3 \int_0^{\sqrt{4-y}} (x+y)\, dx\, dy$ hence evaluate it.

b) Evaluate $\int_0^1 \int_0^x \int_0^{x+y} (x+y+z)\, dz\, dy\, dx$

16. a) State Cayley-Hamilton theorem and using this theorem find the inverse of $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

b) Define i) alternating series ii) Conditional convergence of a series. Also test the Absolute convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2-1}$

17. Answer any **two** of the following:

a) Do the functions $u = \frac{x}{y}; v = \frac{x+y}{x-y}$ functionally dependent? If so, find the relationship between them.

b) Define i) radius of curvature ii) circle of curvature iii) centre of curvature of a point P on a curve y = f(x). Also define the Evolute of a given curve 'C'.

c) Draw a rough sketch of the region of integration $\int_{-1}^{4} \int_{x^2-10}^{3x-6} f(x,y)$.

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